OLLSCOIL LUIMNIGH
UNIVERSITY OF LIMERICK

MSSI

Treatment of Constraint in Non-Linear Fracture Mechanics

Noel O'Dowd
Department of Mechanical and Aeronautical Engineering
Materials and Surface Science Institute
University of Limerick
Ireland

Acknowledgements: C. Fong Shih, S. Kamel. J. Sawyer, H. McGillivray, T. Tkazcyk

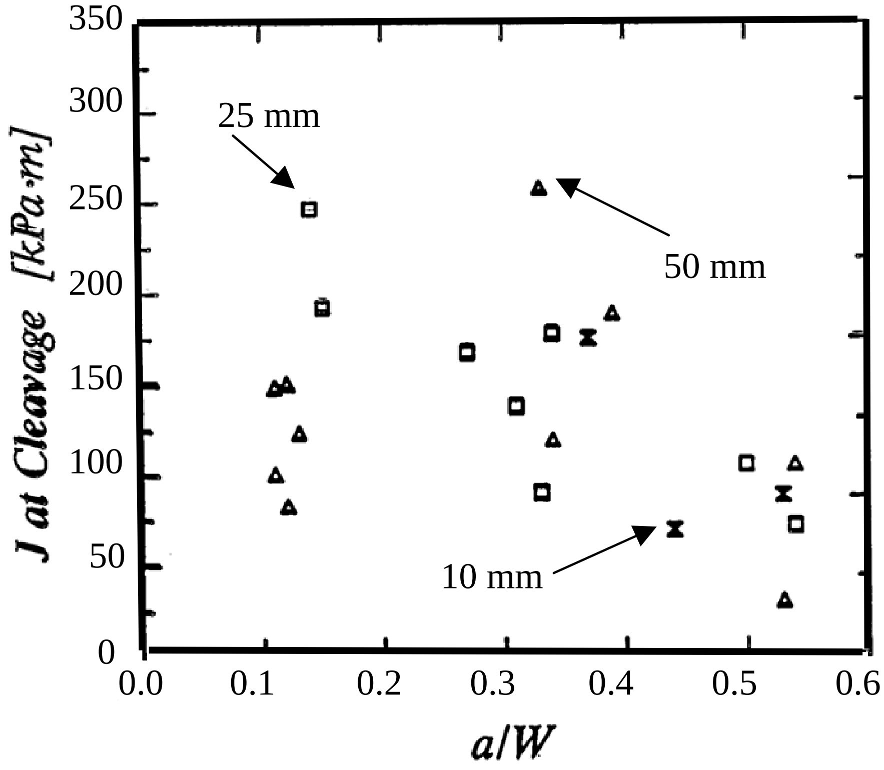
Bristol UK, June 20th 2008

Agenda

- Motivation and Background
- Discussion of higher order terms in crack tip fields
- Application to idealised materials and geometries
- Application to real conditions

Motivation

- Variation in measured fracture toughness for ductile materials



ECB = edge cracked bend
a: crack length
W: specimen width

ASTM A515 from ECB specimens
Kirk et al. 1991

- Toughness (J_C) can depend on specimen geometry and size

Motivation

- J does not uniquely characterise material toughness
- In standard practice a unique toughness value is ensured by following size and geometry requirements for testing
- However, requirements can lead to a very conservative toughness and this motivated research into extending the J-based approach

HRR field

- Hutchinson, 1968; Rice and Rosengren, 1968

- HRR field is first term in the asymptotic solution for a power law plastic material

$$\varepsilon/\varepsilon_0 = \alpha\left(\sigma/\sigma_0\right)^n$$

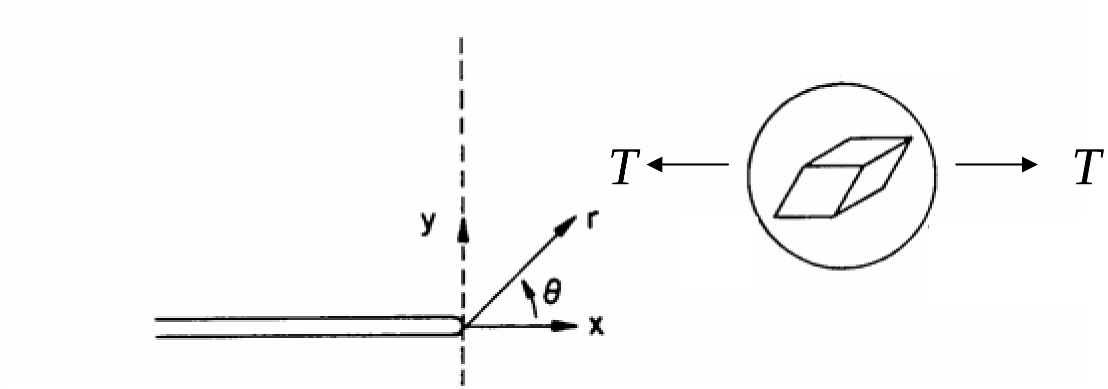
$$\sigma_{ij}/\sigma_0 = \left(\frac{J}{\alpha\varepsilon_0\sigma_0 I_n r}\right)^{1/n+1} \tilde{\sigma}_{ij}(\theta)$$

- Amplitude of HRR stress field is *J*

Motivation

- Two term Williams Mode I crack tip field:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_I(\theta) + T\delta_{ij}$$

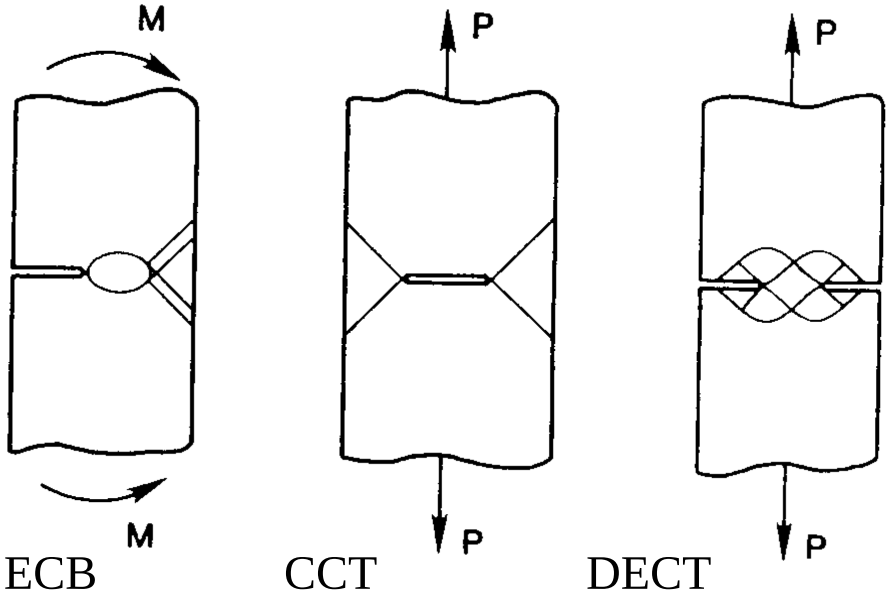


- For a linear elastic material stress term is parallel to the crack face so has weak effect on crack tip driving force

- T stress can effect stability of crack path (Cotterell and Rice, 1980)

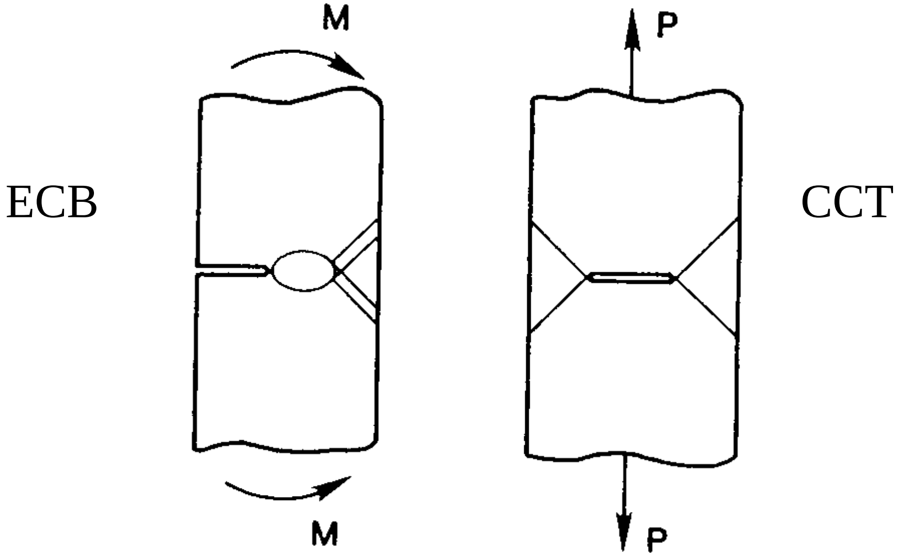
Slip Line Field Solutions

- Slip line field solutions for rigid-perfectly plastic behaviour
- Can identify regions of intense plastic slip—slip bands



- Slip line fields and hence near tip stresses are considerably different

Slip Line Field Solutions



$$\sigma_{22} = \frac{2+\pi}{\sqrt{3}}\sigma_y \qquad \sigma_{22} = \frac{2}{\sqrt{3}}\sigma_y$$

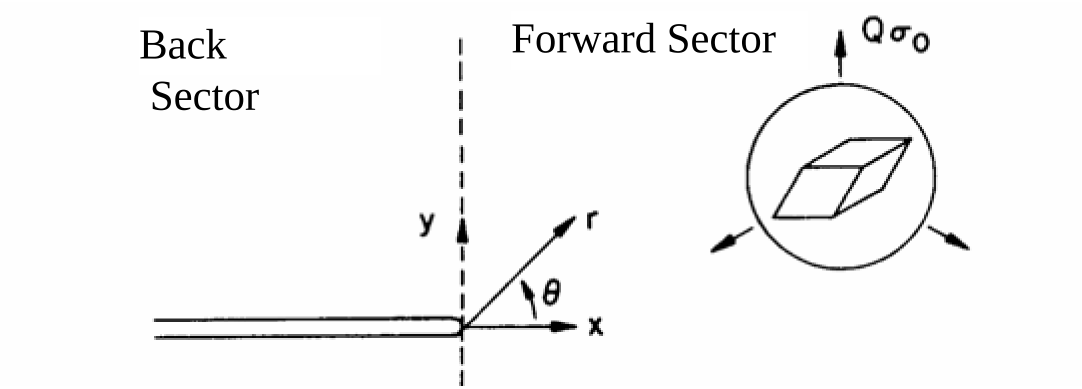
- Slip line fields indicate that bend specimen has higher near tip stress distribution

Constraint in Elastic-plastic Fracture Mechanics

- Crack tip stresses lower in tension than in bending

- For a rigid plastic material can arbitrarily superimpose a hydrostatic stress (provided boundary conditions are satisfied))

- Variation in stress associated with a variation in hydrostatic stress



- This difference in hydrostatic stress is a measure of the difference in ‘constraint’ between different specimen geometries

Constraint in Elastic-plastic Fracture Mechanics

- Crack tip fields studied numerically through “boundary layer” finite element analyses

- Larsson and Carlsson, 1973, used such an approach to examine the effect of T-stress on plastic zone size

- Similar studies by Bilby et al., 1986 and Betegón and Hancock, 1990

- Material model: Elastic-plastic rate independent power law material:

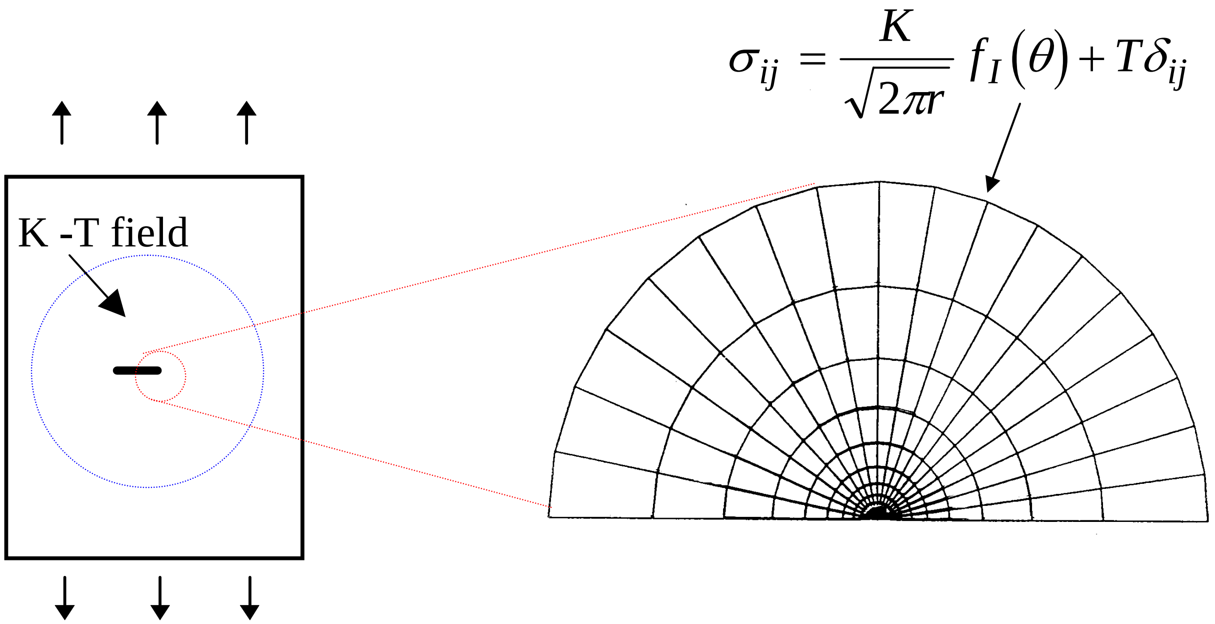
$$\varepsilon/\varepsilon_0 = \sigma/\sigma_0 \qquad \sigma < \sigma_0$$

$$\varepsilon/\varepsilon_0 = (\sigma - \sigma_0)/\sigma_0 + \underset{\uparrow}{(\sigma/\sigma_0)^n}$$

Power law plasticity

Finite Element Boundary Layer Analysis

- Effect of geometry introduced through T stress term applied at the boundary

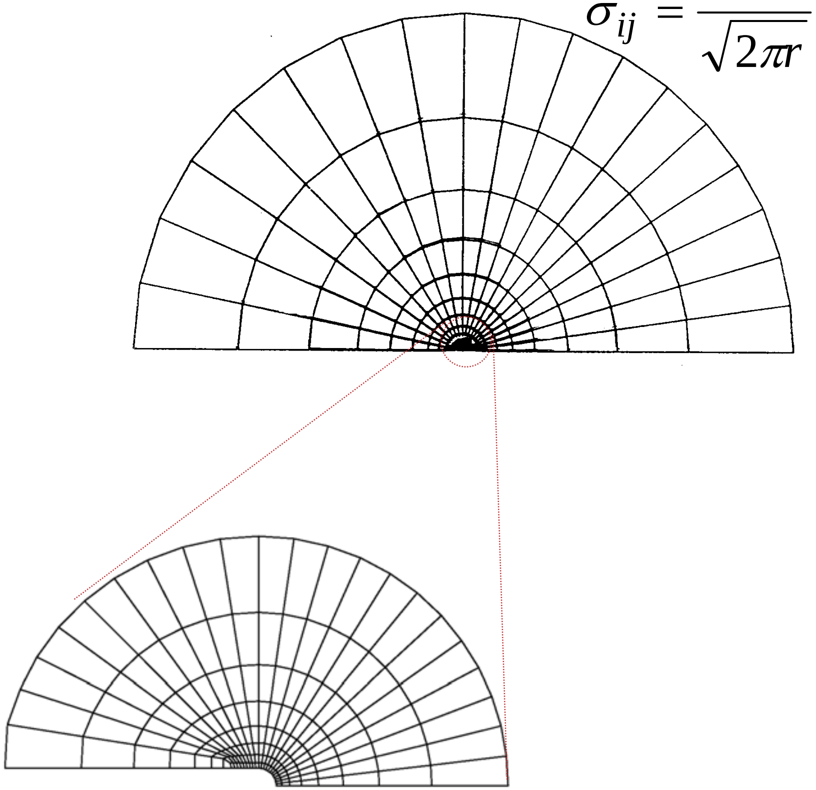


$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_I(\theta) + T\delta_{ij}$$

- When T =0 the analysis is often referred to as a small scale yielding analysis (SSY) and the resultant stress field is the SSY field

Finite Element Boundary Layer Analysis

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_I(\theta) + T\delta_{ij}$$

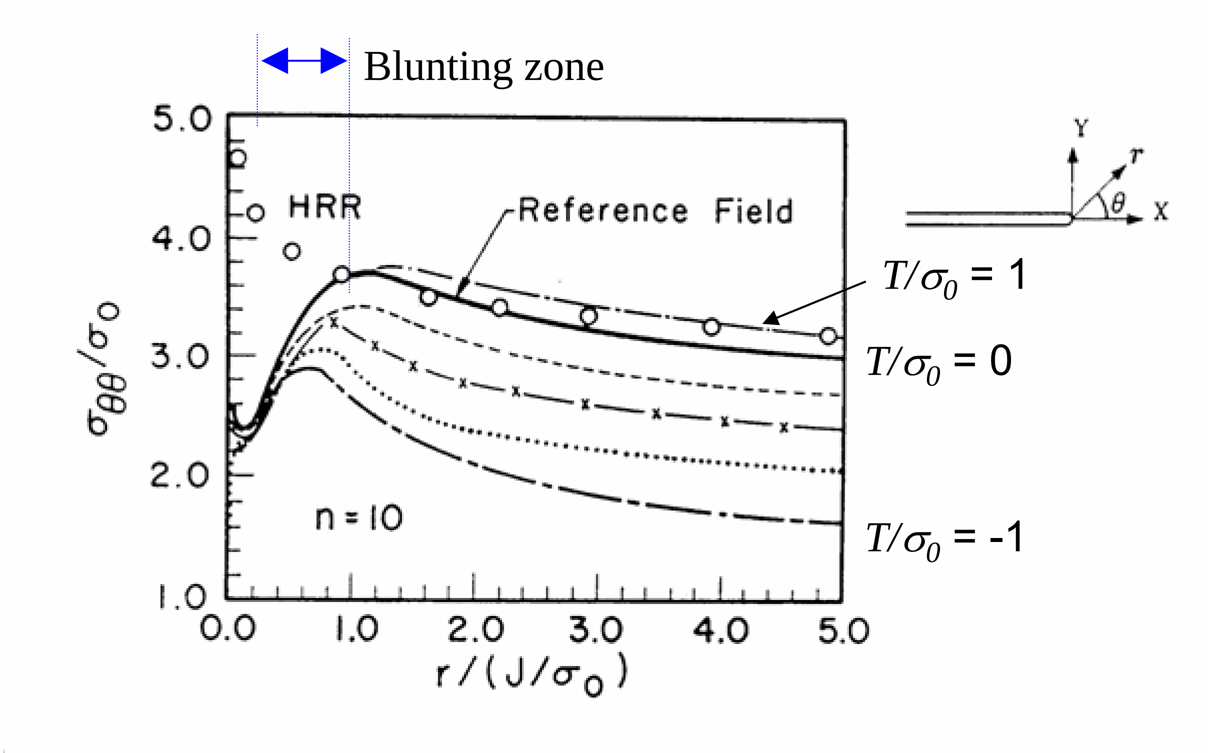


- Finite notch modelled (rather than sharp crack tip) to allow investigation of crack blunting

Finite Element Boundary Layer Analysis

- The T stress plays the role of a geometry parameter
- By varying T/σ_0 can generate a range of crack tip fields
- $T/\sigma_0 = 0$ corresponds (almost) to the HRR field
- Finite strain analysis carried out to account for large deformations in the vicinity of the crack tip

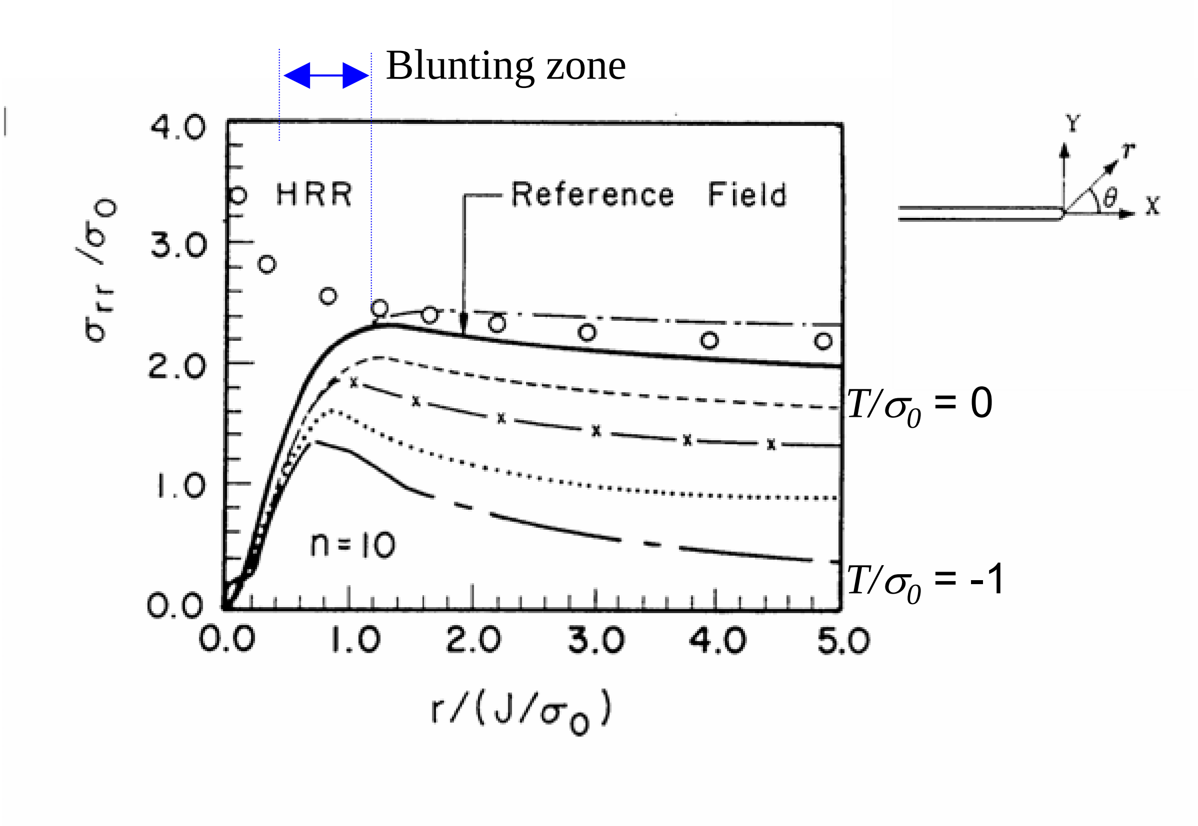
Finite Element Boundary Layer Analysis



- Stresses normalised by σ_0; Distances normalised by J/σ_0

- J/σ_0 is a measure of the crack tip opening and the crack blunting zone

Finite Element Boundary Layer Analysis



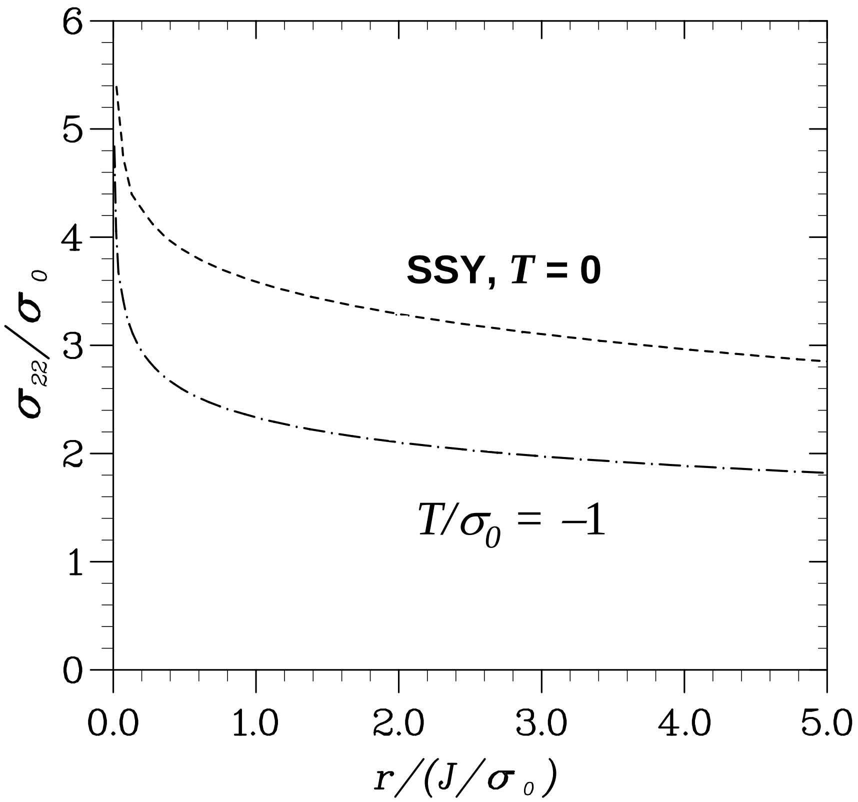
Blunting zone

T/σ_0 = 0

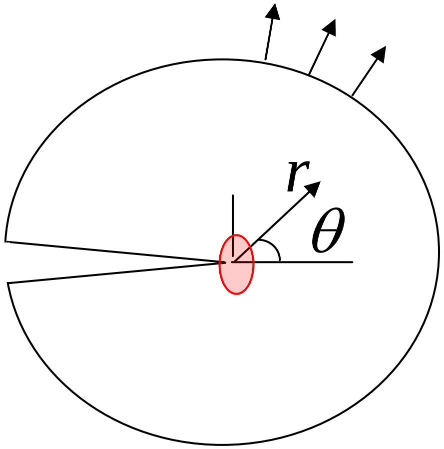
T/σ_0 = -1

- Similar behaviour for radial stress, σ_{rr}

Small strain analysis (no crack blunting)



$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_I(\theta) + T\delta_{ij}$$



Construction of *Q*-stress fields

- From numerical results can construct the form of the second order elastic-plastic stress field

- Define

$$Q\tilde{f}\left(r,\theta\right)=\frac{\sigma_{ij}^{FE}}{\sigma_0}-\frac{\sigma_{ij}^{REF}}{\sigma_0}$$

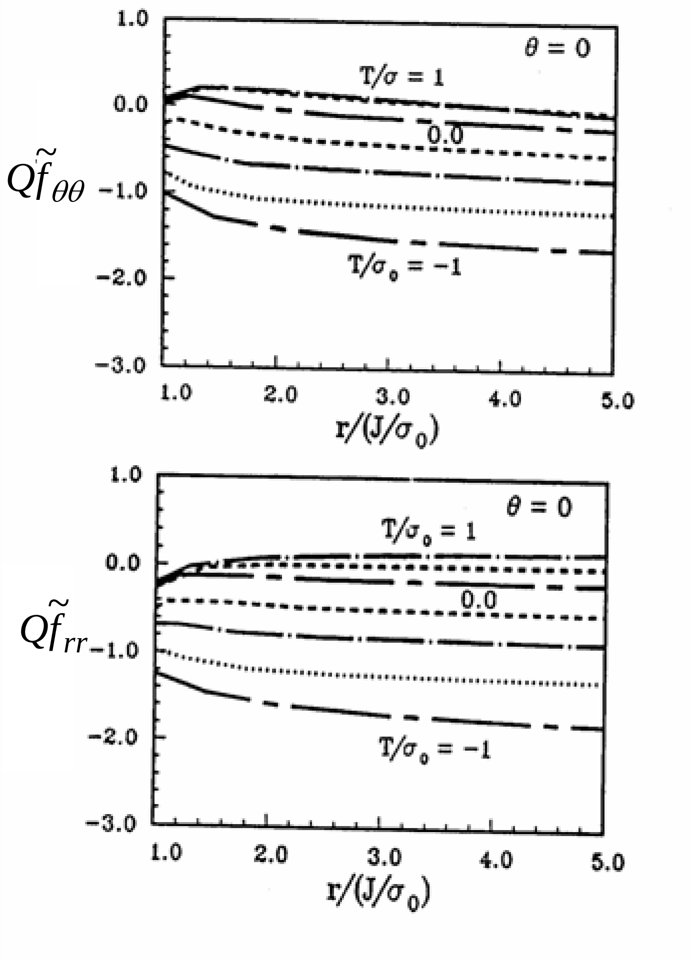
- σ_{REF} is the reference (high constraint) distribution—in this case the SSY (or HRR) field

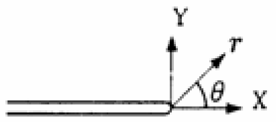
- Q is a dimensionless amplitude parameter of the second order fields

- Q gives the angular and radial distribution of the fields (not yet known)

Construction of Q-stress fields

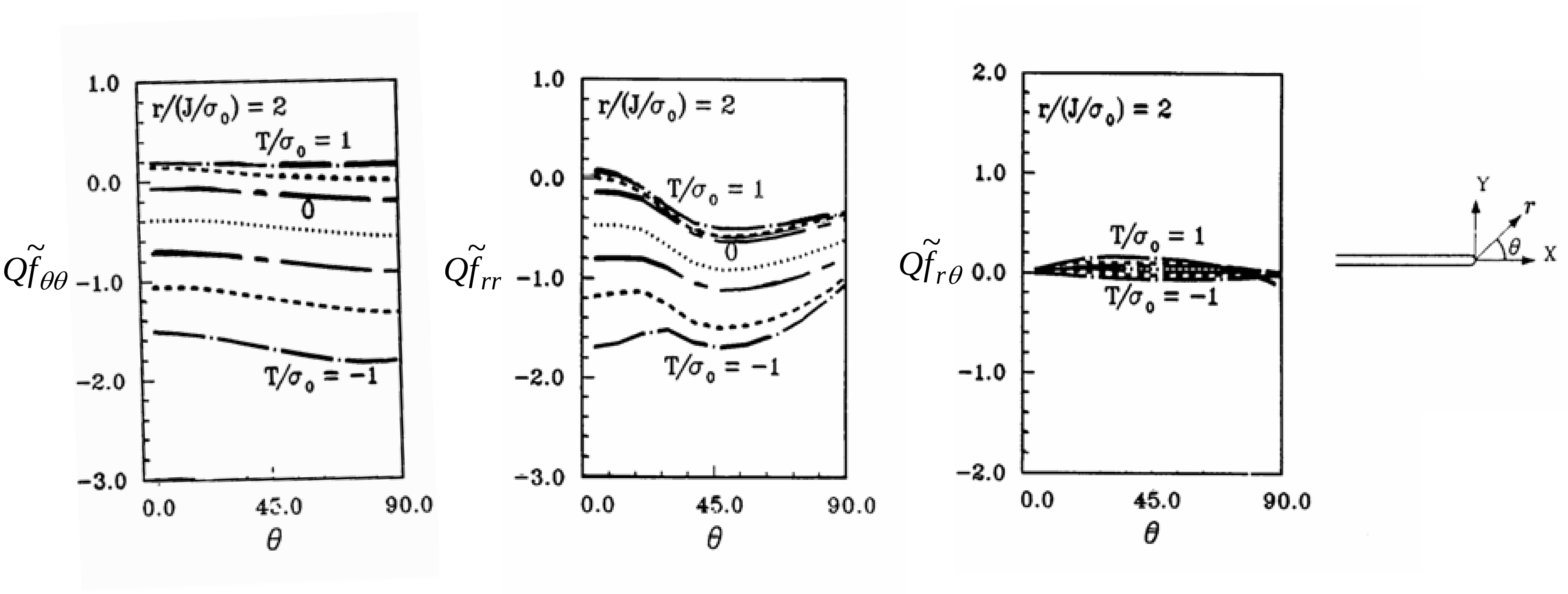
- From numerical results can construct the form of the second order elastic-plastic stress field





Construction of Q-stress fields

- Angular distribution:



- To a good approximation:

$$\tilde{f}_{\theta\theta}(r,\theta) = \tilde{f}_{rr}(r,\theta) = 1$$

$$\tilde{f}_{r\theta}(r,\theta) = 0$$

Two parameter elastic-plastic stress fields

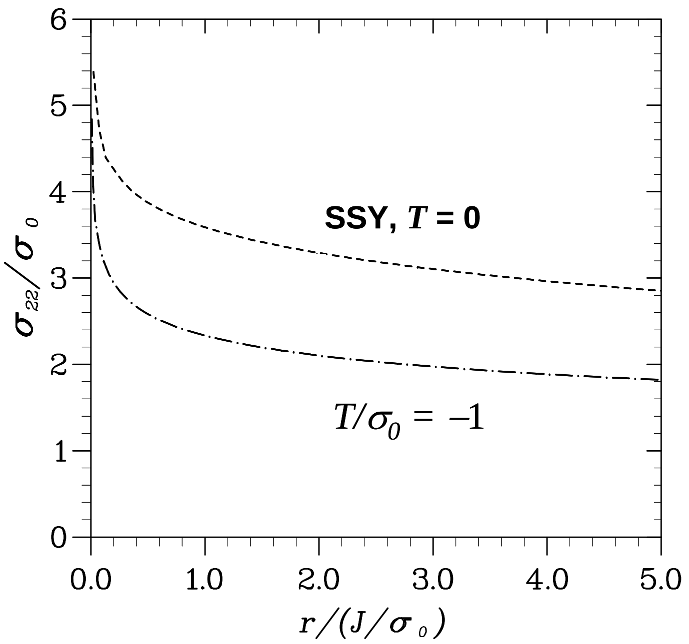
$$\sigma_{ij} / \sigma_0 = \left(\frac{J}{\alpha \varepsilon_0 \sigma_0 I_n r} \right)^{1/n+1} \tilde{\sigma}_{ij}(\theta) + Q\delta_{ij}$$

- or, more generally

$$\sigma_{ij} / \sigma_0 = \left(\frac{\sigma_{ref}}{\sigma_0} \right) + Q\delta_{ij}$$

- The parameter Q is a hydrostatic stress term determined from finite element analysis for the particular geometry and load level

Two parameter elastic-plastic stress fields



- Q will generally be negative—HRR or SSY field is the upper bound 'high constraint' crack tip field
- Q reduces the stress amplitude relative to the reference field

Two Parameter Fracture Mechanics

$$\sigma_{ij} / \sigma_0 = \sigma_{ref} / \sigma_0 + Q\delta_{ij}$$

- Stress and strain fields depend on J and Q
- Fracture toughness expressed in terms of $J_C(Q)$
- J_{IC} is the standard high constraint fracture toughness value corresponding to $Q = 0$ and is generally the lower bound

Asymptotic solution, (Yang *et al.*, 1993)

- Analytical solution for power-law material model.
- First three terms of crack tip field:

$$\frac{\sigma_{ij}}{\sigma_0} = \left(\frac{J}{\alpha\varepsilon_0\sigma_0 I_n r}\right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}^{HRR} + A\left(\frac{J}{\alpha\varepsilon_0\sigma_0 I_n L}\right)^{\frac{1}{n+1}}\left(\frac{r}{L}\right)^{s} \tilde{\sigma}_{ij}^{(1)} + A^2\left(\frac{J}{\alpha\varepsilon_0\sigma_0 I_n L}\right)^{\frac{1}{n+1}}\left(\frac{r}{L}\right)^{t} \tilde{\sigma}_{ij}^{(2)}$$

$\tilde{\sigma}_{ij}$ are functions of n and θ

Exponents s and t depend on n

L is a characteristic length (*e.g.* unity, a, W)

- Crack tip stress fields are described by J and A.
- A is a measure of the loss of constraint (equivalent to Q)

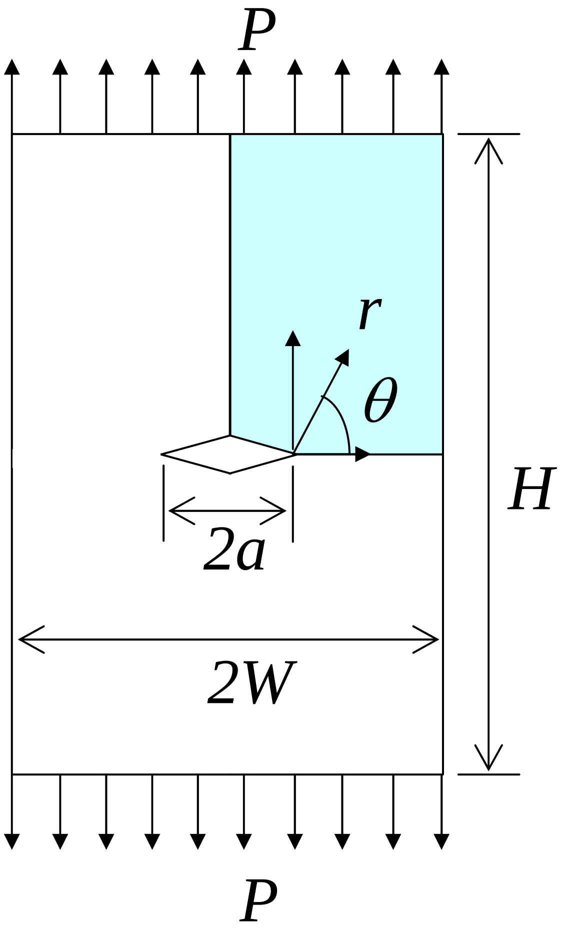
Two Parameter Fracture Mechanics

- Analysis so far has been for idealised geometry, boundary layer analysis
- Now consider ‘real’ geometries

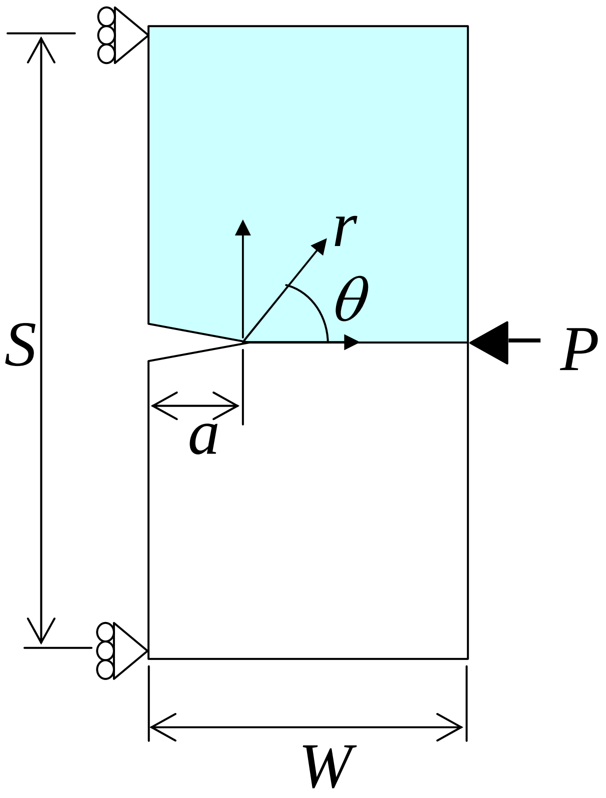
FE models

- All models discussed are two dimensional

M(T)



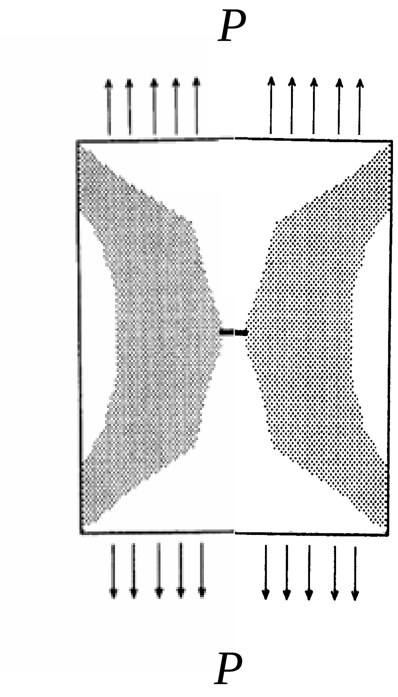
SEN(B)

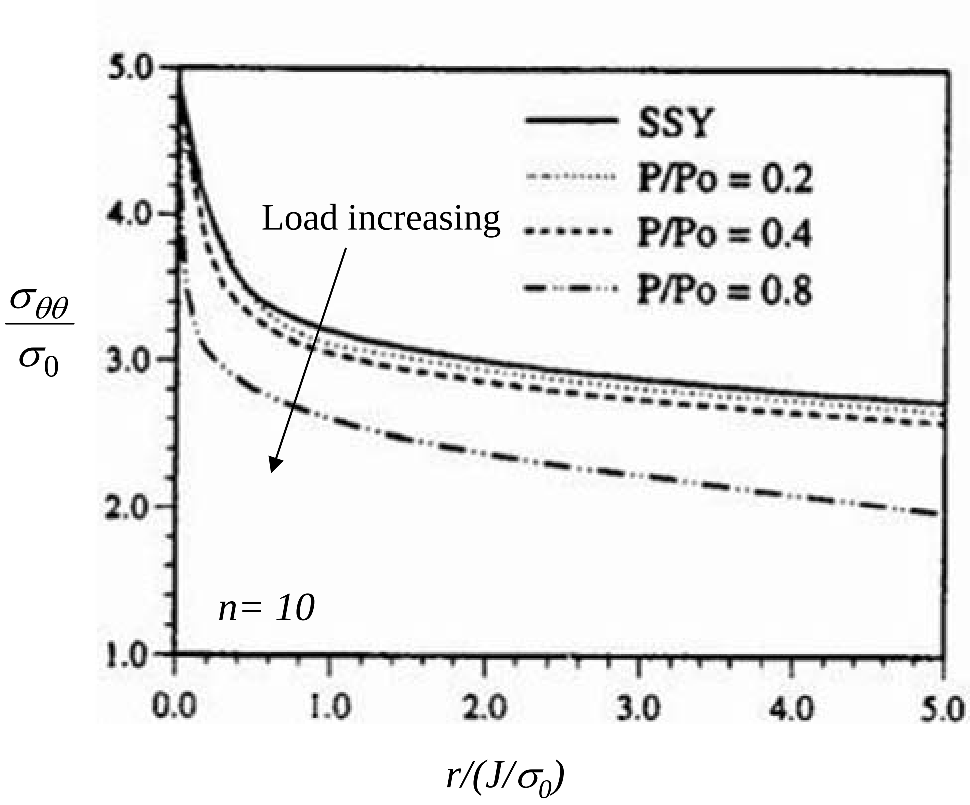


- Crack depths modelled: $a/W = 0.1 \rightarrow 0.7$

- Gives expected range of constraint in practice.

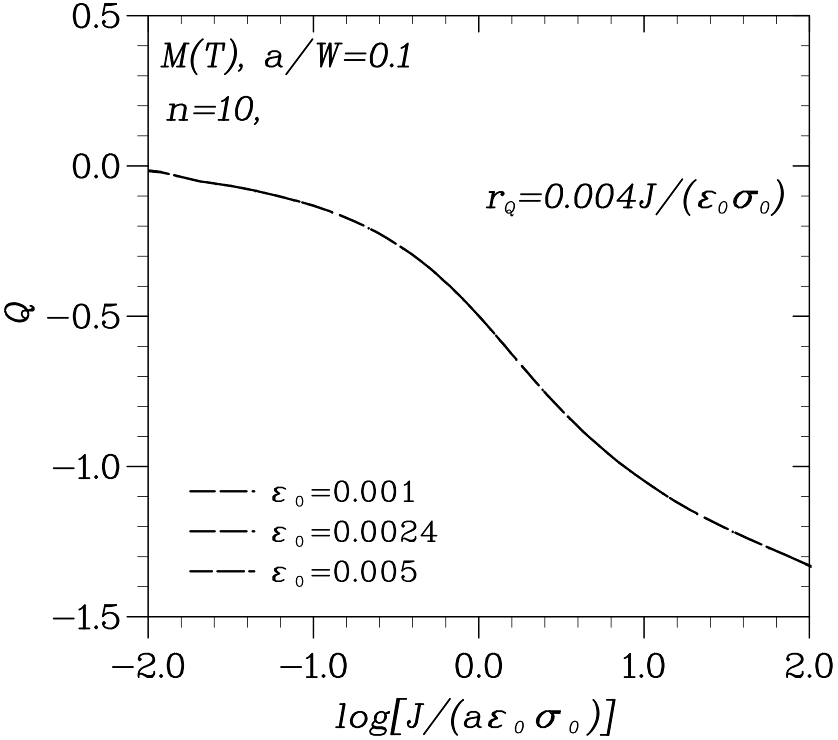
Results for Centre-Cracked Tension





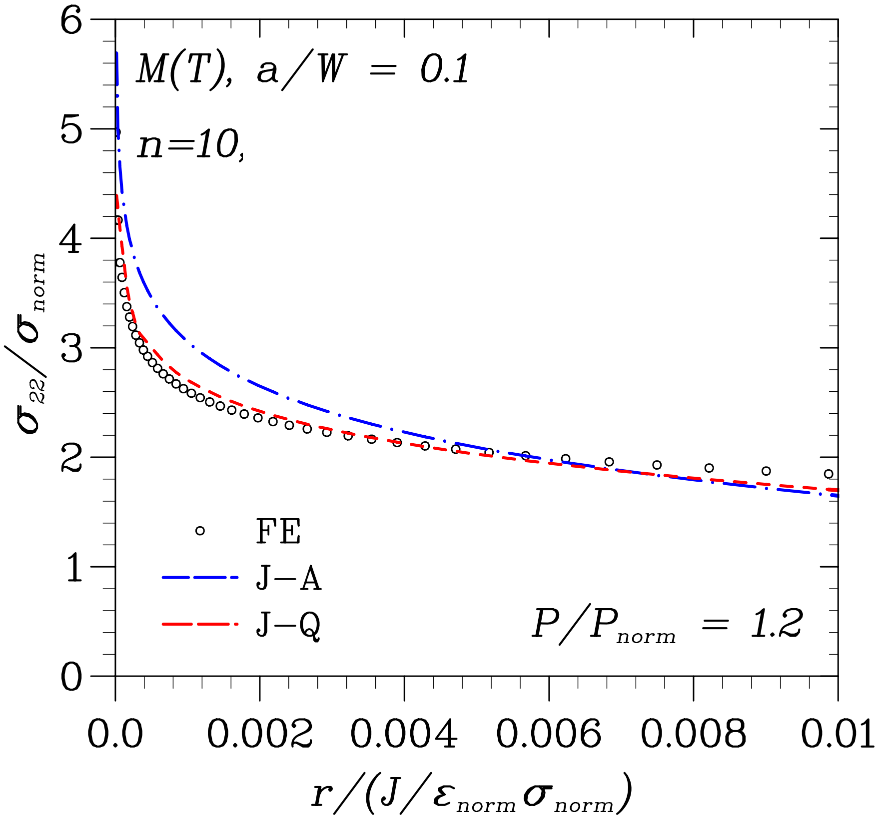
- Constraint drops with increasing load—*Q* becomes more negative

Dependence of *Q* on normalised *J*



- Q is evaluated at $r_Q = 0.004\, J/(\varepsilon_0\sigma_0)$
  - corresponds to $r_Q = 2J/\sigma_0$ for $\varepsilon_0 = 0.002$ and $\alpha = 1$

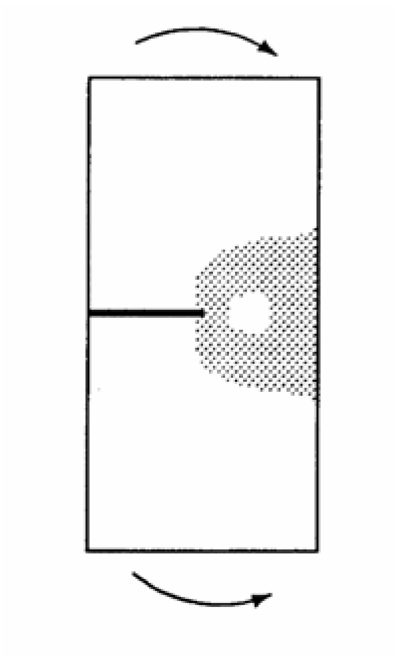
Analysis of M(T), *a/W* = 0.1

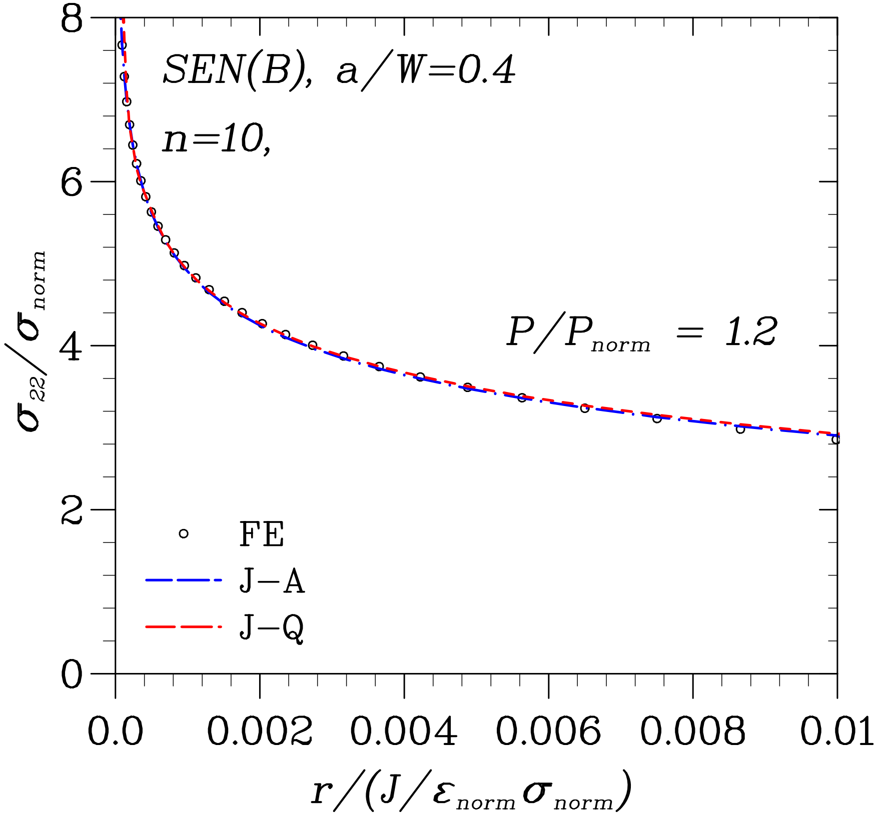


$$P_{norm} = \frac{4b}{\sqrt{3}}\sigma_{norm}$$

- *J-Q* gives a somewhat better prediction for *n* = 10 and 5
- Representative of M(T), *a/W*=0.4 and 0.7 and SEN(B), *a/W*=0.1

Analysis of SEN(B), a/W = 0.4, n = 10

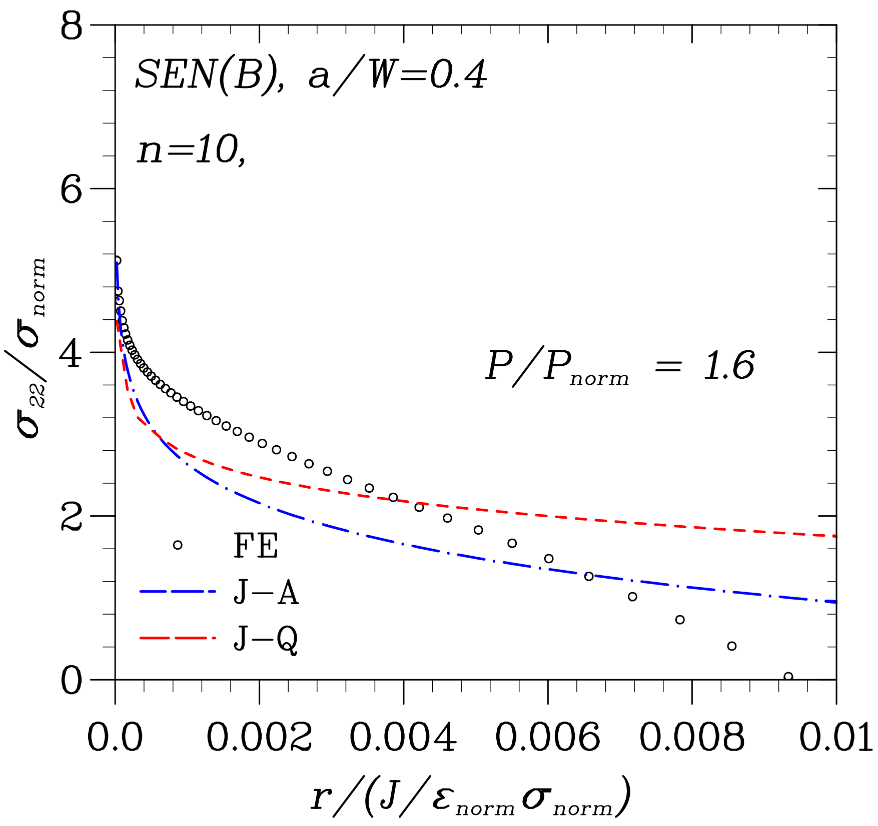




$$P_{norm} = \frac{1.409 b^2 \sigma_{norm}}{S}$$

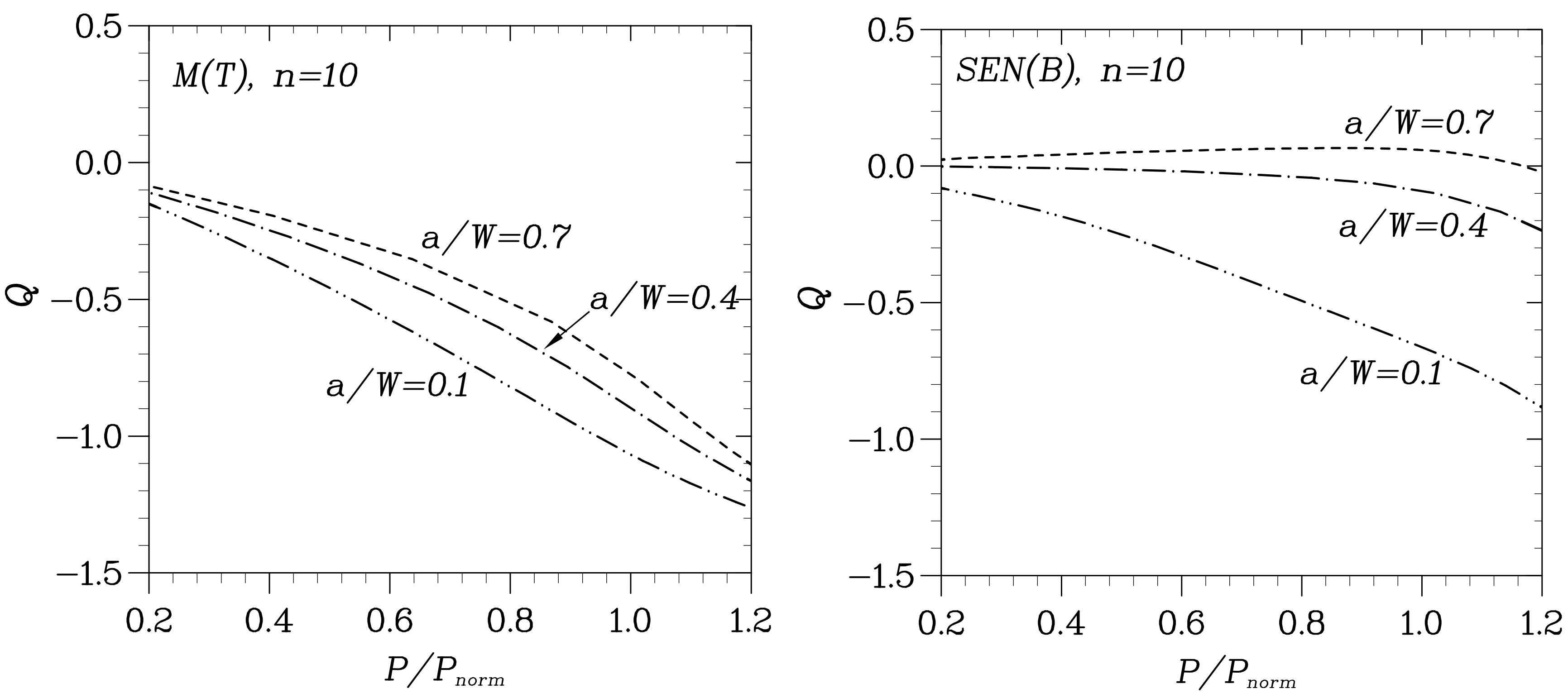
- Both J-Q and J-A give excellent agreement with FE prediction at this deformation.

Analysis of SEN(B), a/W = 0.4, n = 10



- At higher deformation both J-Q and J-A give poor prediction due to effect of global bending on crack tip fields.
- Global bending may be characterised by an additional parameter (Chao *et. al.*, 2004; Zhu & Lei, 2006).

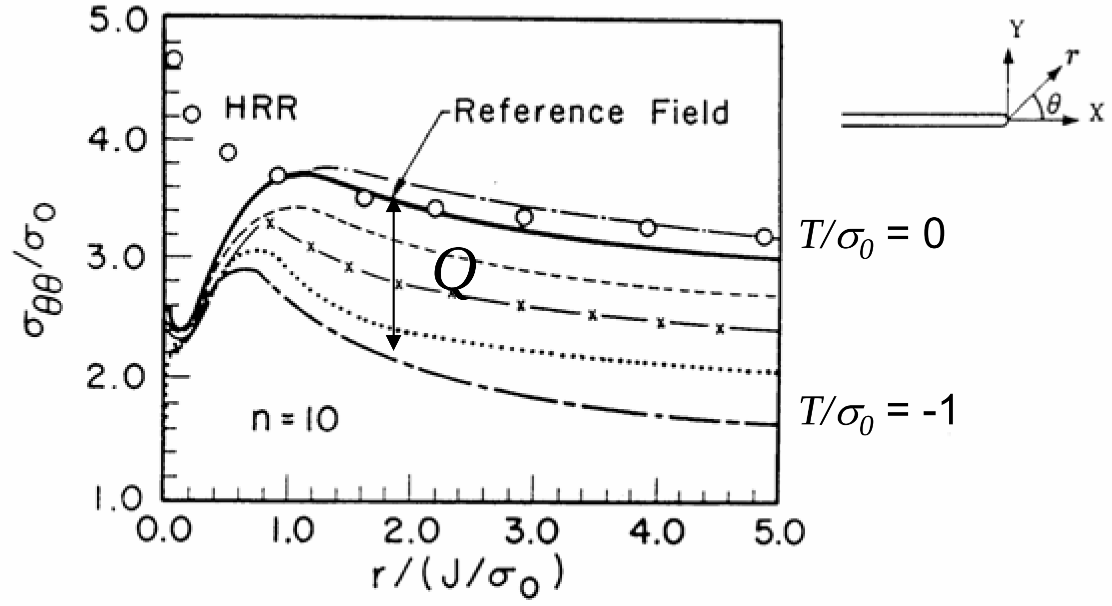
Q vs Load



- Tension geometries and shallow cracked bend geometry experience ‘loss of constraint’ at low load
- Slope of Q vs load is approx. constant at high deformation

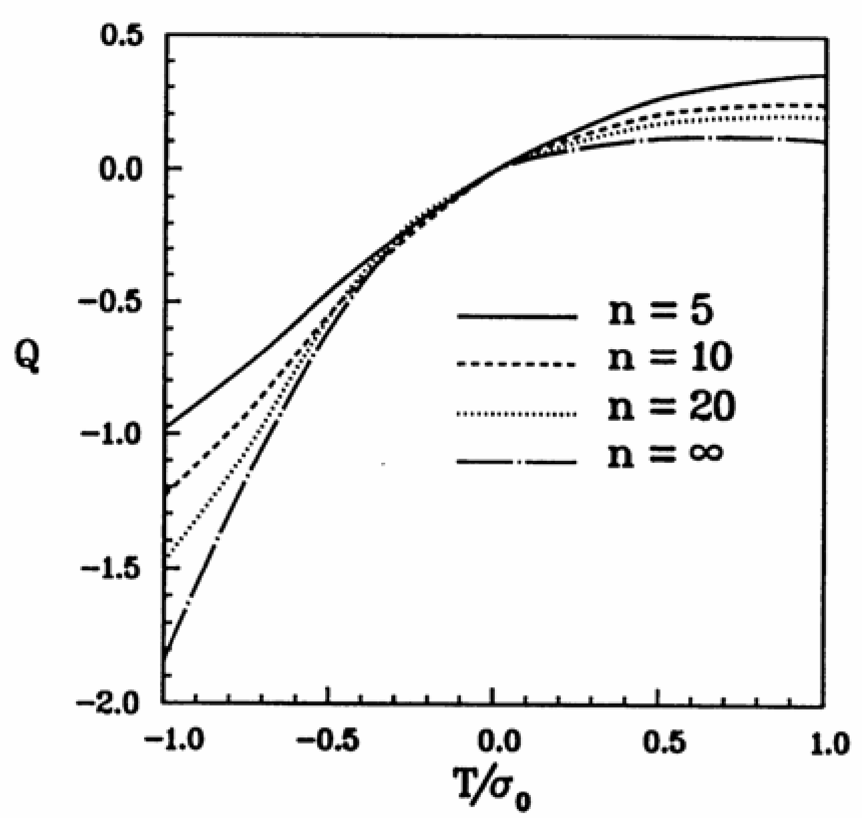
Evaluation of *Q*

- Within the boundary layer analysis there is a one-to-one relationship between T and Q for a given material



Evaluation of Q

- T is an elastic parameter and is relatively easily determined
- Can thus estimate Q through an elastic analysis for T

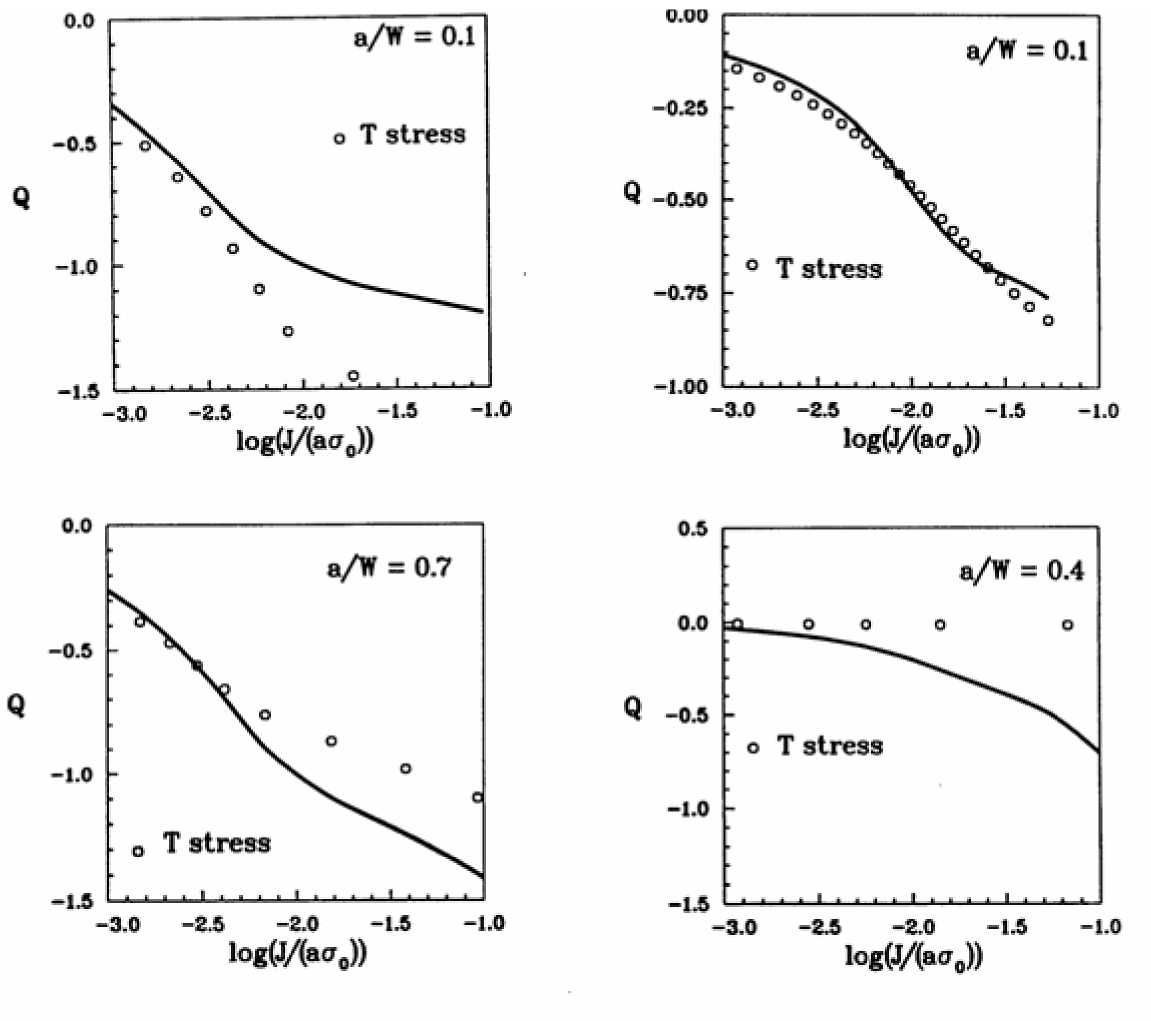


Evaluation of Q

- T stress can give a reasonable estimate of Q

Centre Cracked Tension

Edge Cracked Panel



Power law estimates for Q

- T-stress can be used to estimate Q under ‘small scale yielding’ conditions—when size of the plastic zone is small
- Consider a pure power law material:

$$\varepsilon / \varepsilon_0 = \alpha\left(\sigma / \sigma_0\right)^n$$

- For such a material, stress at a point varies linearly with remote load

$$\sigma_{ij} / \sigma_0 = \left(\sigma^{\infty} / \sigma_0\right)\bar{\sigma}_{ij}\left(x,n\right)$$

Power law estimates for Q

- It can be shown using the HRR field that

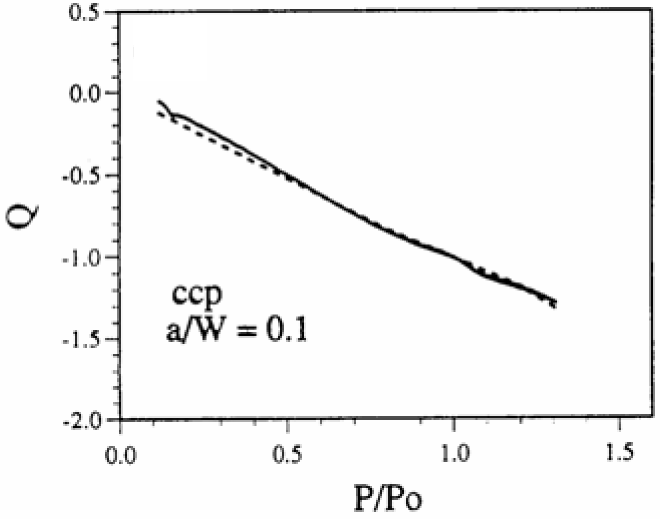
$$\frac{J}{\alpha \varepsilon_0 \sigma_0 a} = \left(\sigma^\infty / \sigma_0\right)^{n+1} h_1(n)$$

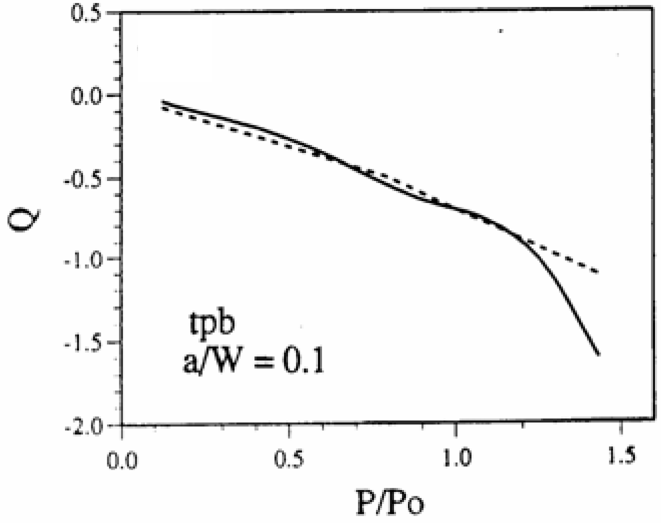
- h_1 is a function which depends only on geometry and n
- Similarly it can be shown for a pure power law material

$$Q = \left(\sigma^\infty / \sigma_0\right) \, h_2(n)$$

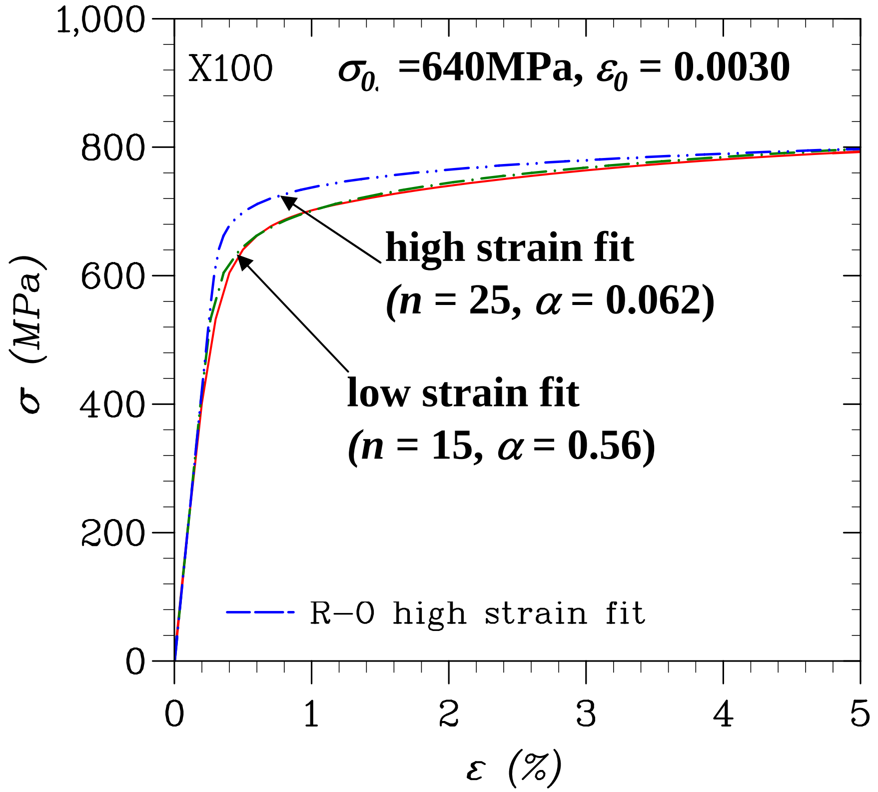
- Q varies linearly with load
- An approximation scheme based on T under small scale yielding and h_2 under extensive plasticity may then be used

Power law estimates for *Q*



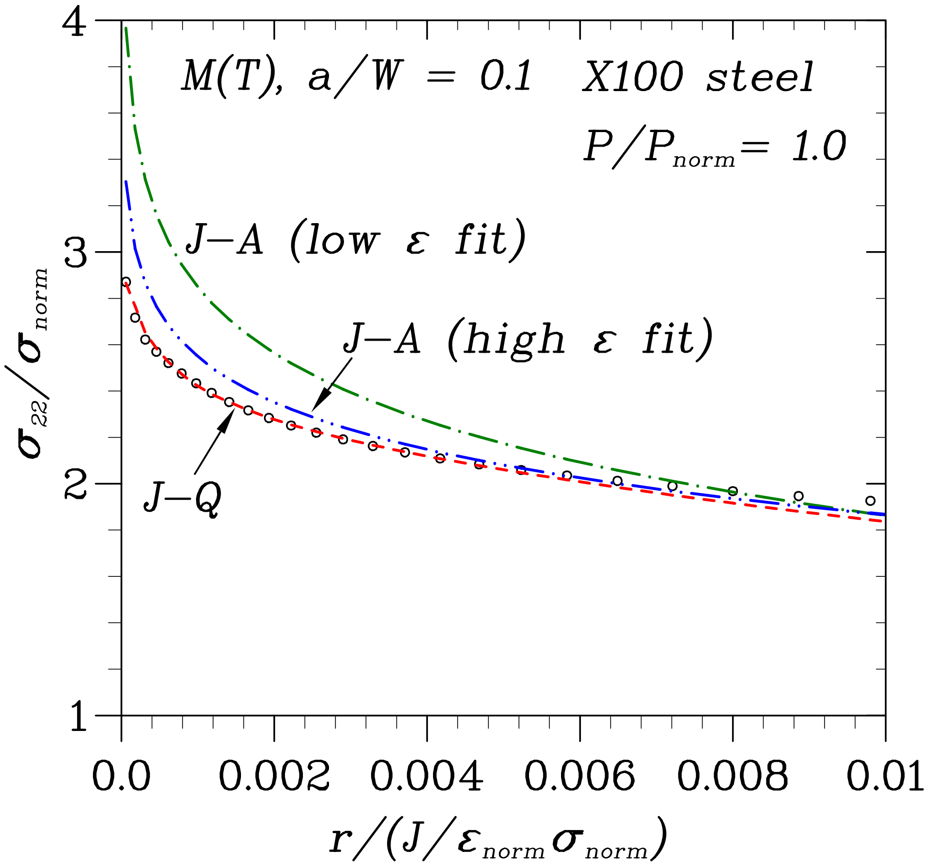


Treatment of 'real' materials: X100 ferritic pipeline steel



- Low strain fit with n =15 gives good fit to test data up to 5% strain.
- High strain fit with n =25 gives close fit at strains > 5%

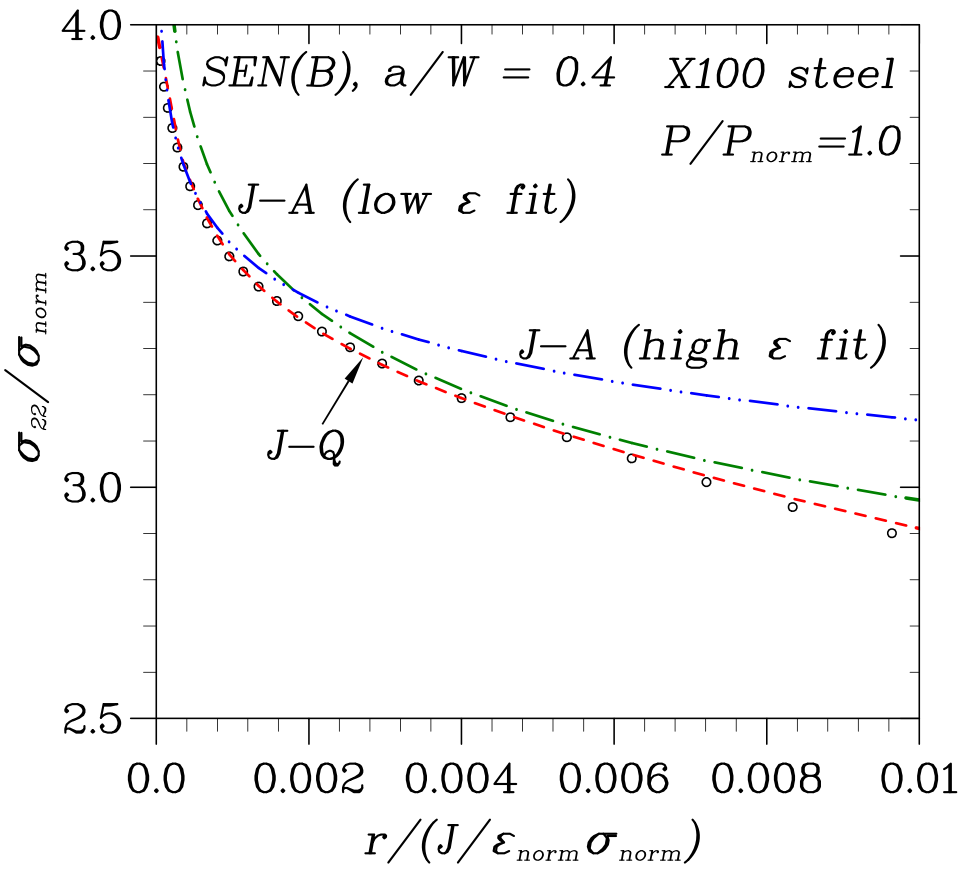
Results for X100: M(T), *a/W* =0.1



Kamel et al., 2007

- *J-Q* gives best agreement with FE.
- *J-A* prediction based on *high* strain fit gives better agreement than the *low* strain fit.

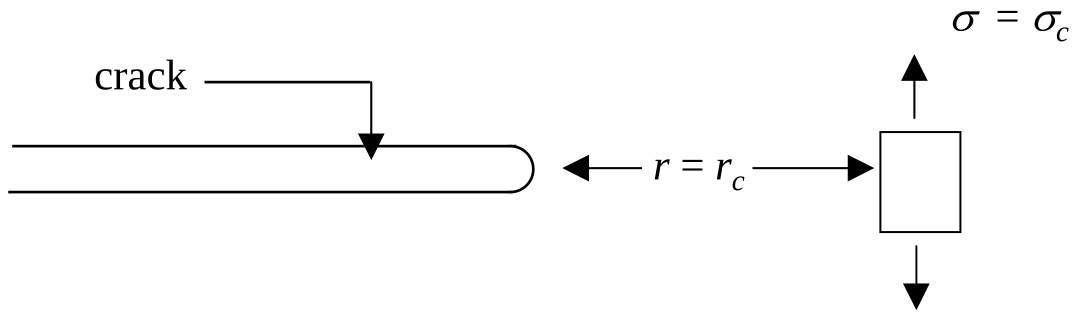
Results for X100: SEN(B), *a/W* =0.4



- *J*-*Q* gives best agreement with FE.
- *J*-*A* prediction based on *low* strain fit gives better agreement than the *high* strain fit.

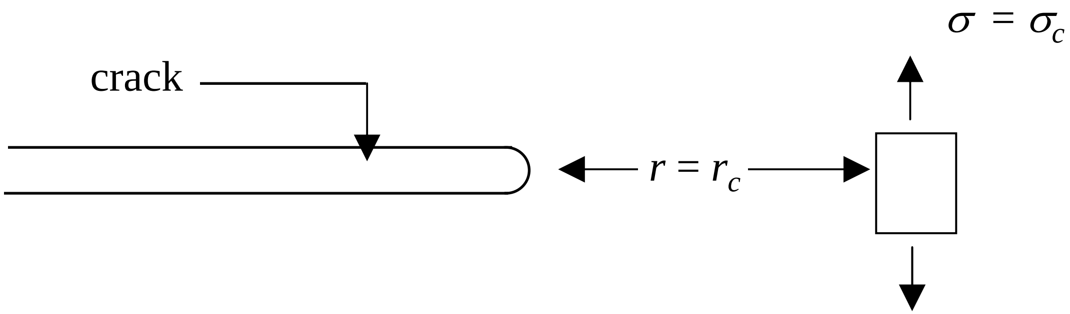
A Simple Fracture Toughness Curve—RKR Model

- Examine fracture criterion based on the attainment of a critical stress at a critical distance



- One parameter fracture toughness gives a single number J_{IC}
- Using two parameter fracture mechanics generate a fracture toughness curve, $J_C(Q)$

A Simple Fracture Toughness Curve—RKR Model



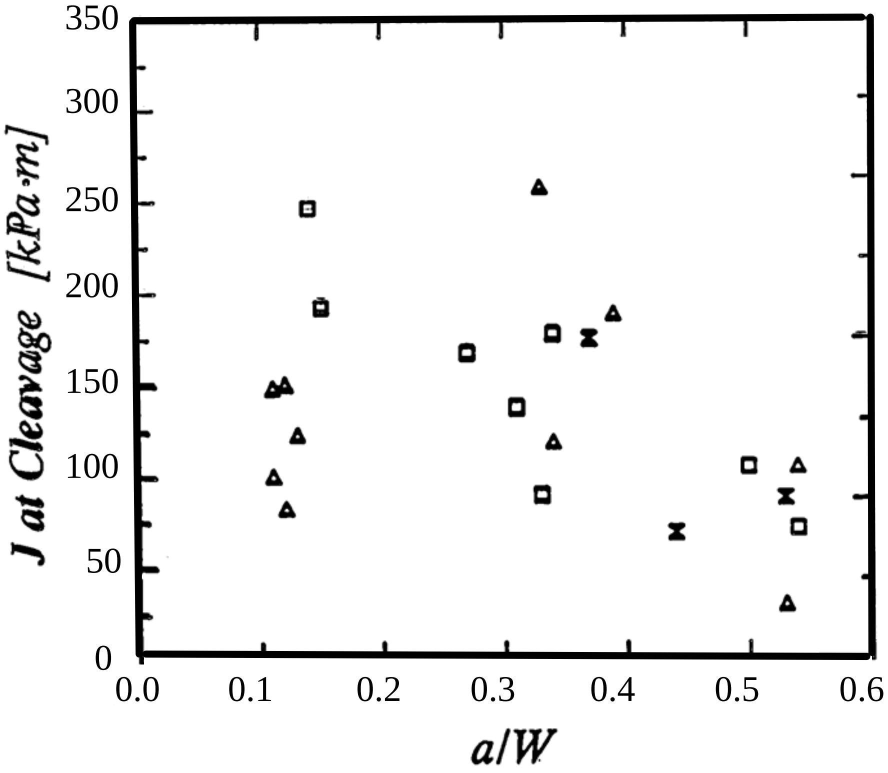
$$\sigma_{22}/\sigma_0 = \left(\frac{J}{\sigma_0\varepsilon_0 I_n r}\right)^{1/n+1} + Q$$

$$\Rightarrow \sigma_c/\sigma_0 = \left(\frac{J_C}{\sigma_0\varepsilon_0 I_n r_c}\right)^{1/n+1} + Q$$

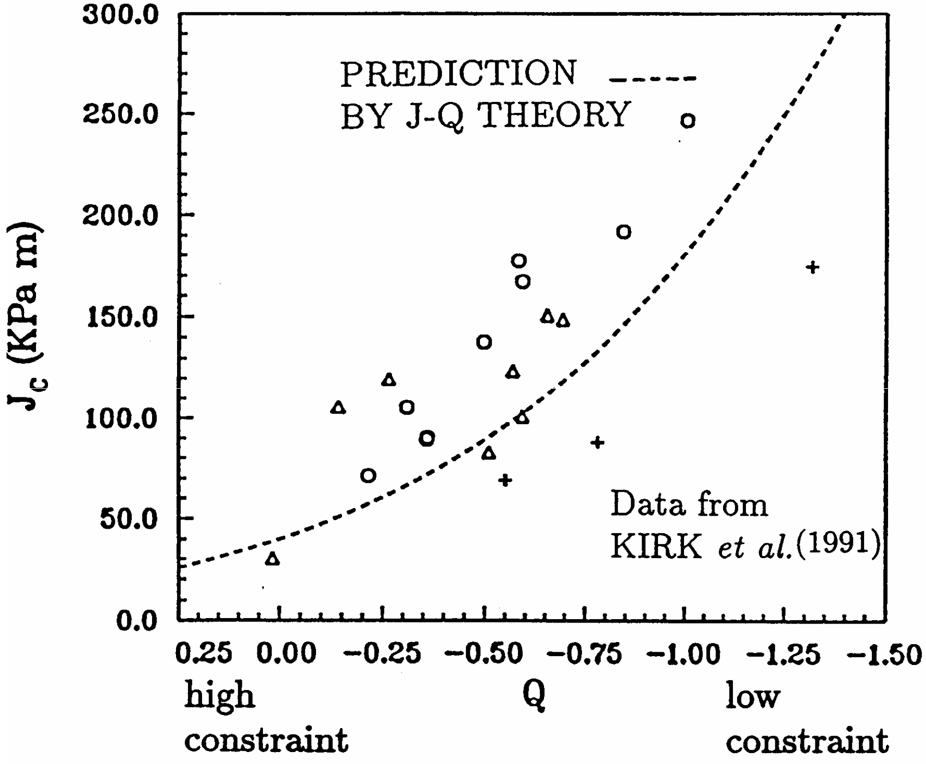
$$Q = 0 \quad \Rightarrow \quad J = J_{IC}$$

$$\Rightarrow \frac{J_C}{J_{IC}} = \left(\frac{\sigma_c/\sigma_0 - Q}{\sigma_c/\sigma_0}\right)^{n+1}$$

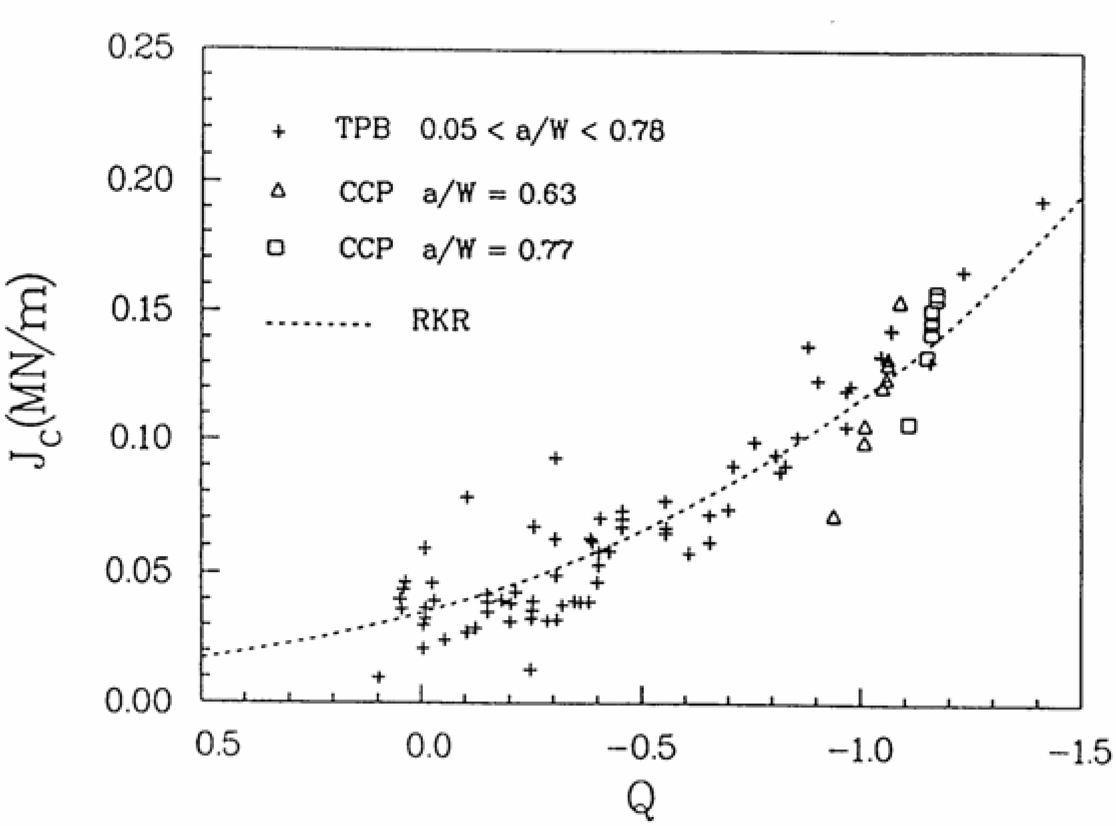
A Simple Fracture Toughness Curve—RKR Model



ASTM A515 from ECB specimens
*Kirk et al.*1991

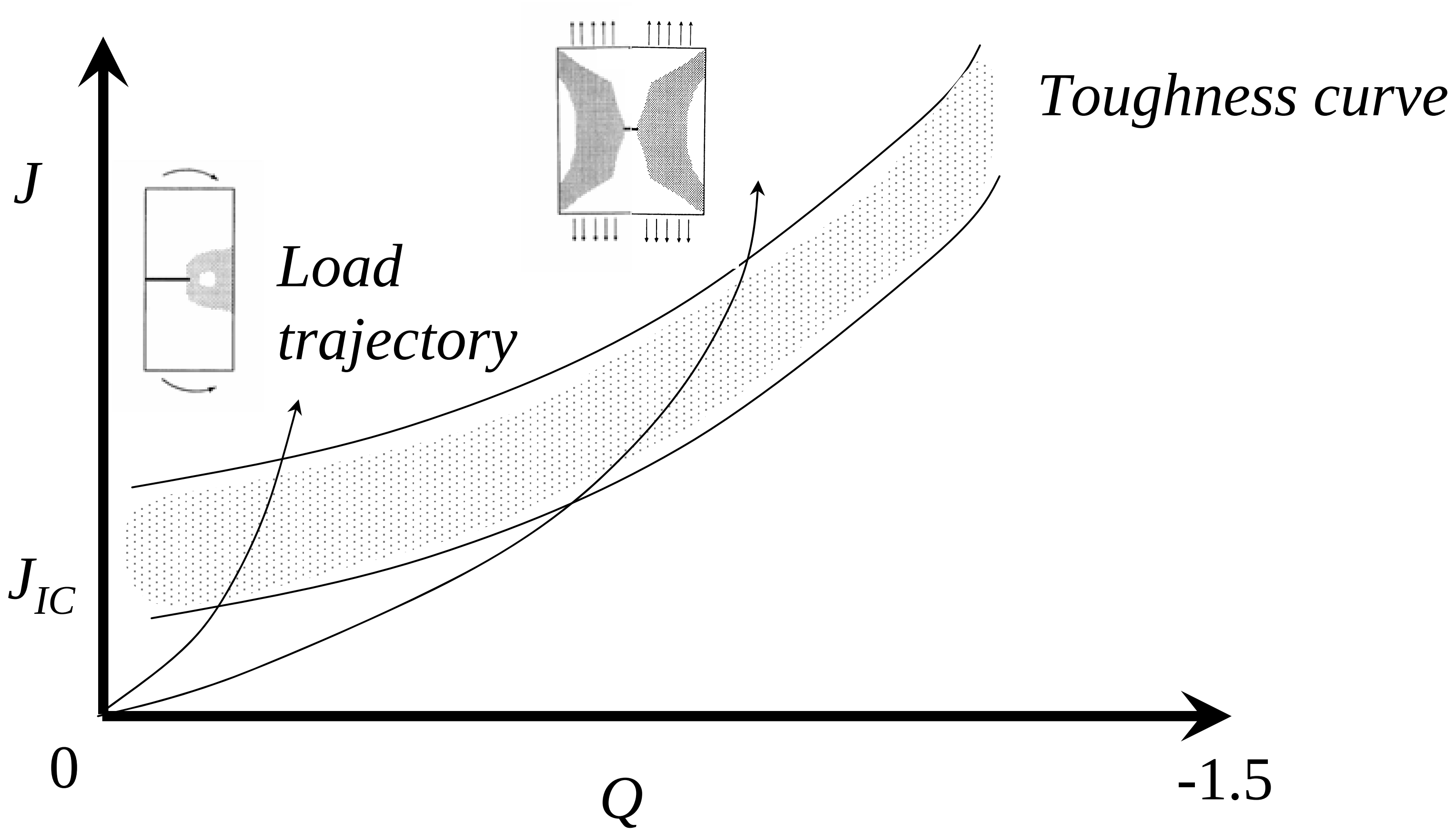


A Simple Fracture Toughness Curve—RKR Model



Cleavage toughness data for mild steel tested at-50^{o}C, Sumpter and Forbes, 1992

Application of *J*-*Q* approach



- Toughness curve determined in laboratory
- J-Q loading trajectory for component from finite-element analysis

Alternative approach: Constraint matching

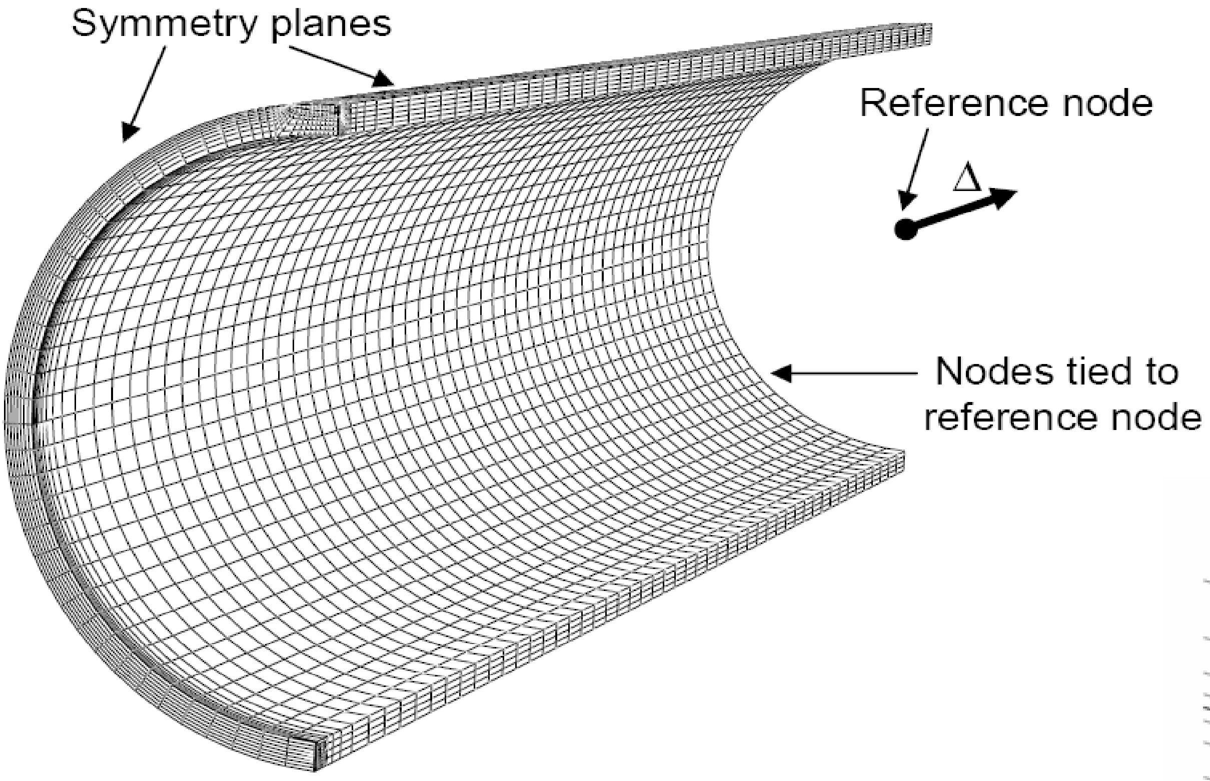
- Estimate Q value at fracture for component
- Test laboratory specimen with similar constraint level
- Treat this fracture toughness as the ‘constraint’ matched toughness
- E.g. For shallow cracked pipes under tension ($Q \approx -1$) use fracture toughness J_c (or CTOD) from edge crack tension geometry
- Approach widely used in offshore industry

Validation for ‘real’ materials

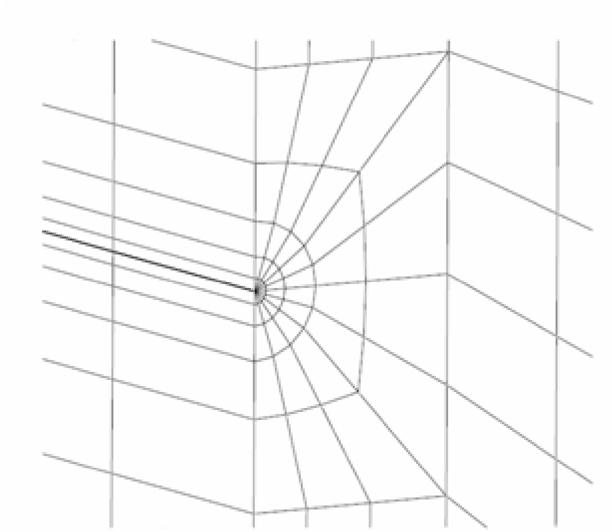
- ☐ Analysis of X65/X70/X100 ferritic pipeline steels

Technip

Approach for Steel Pipelines



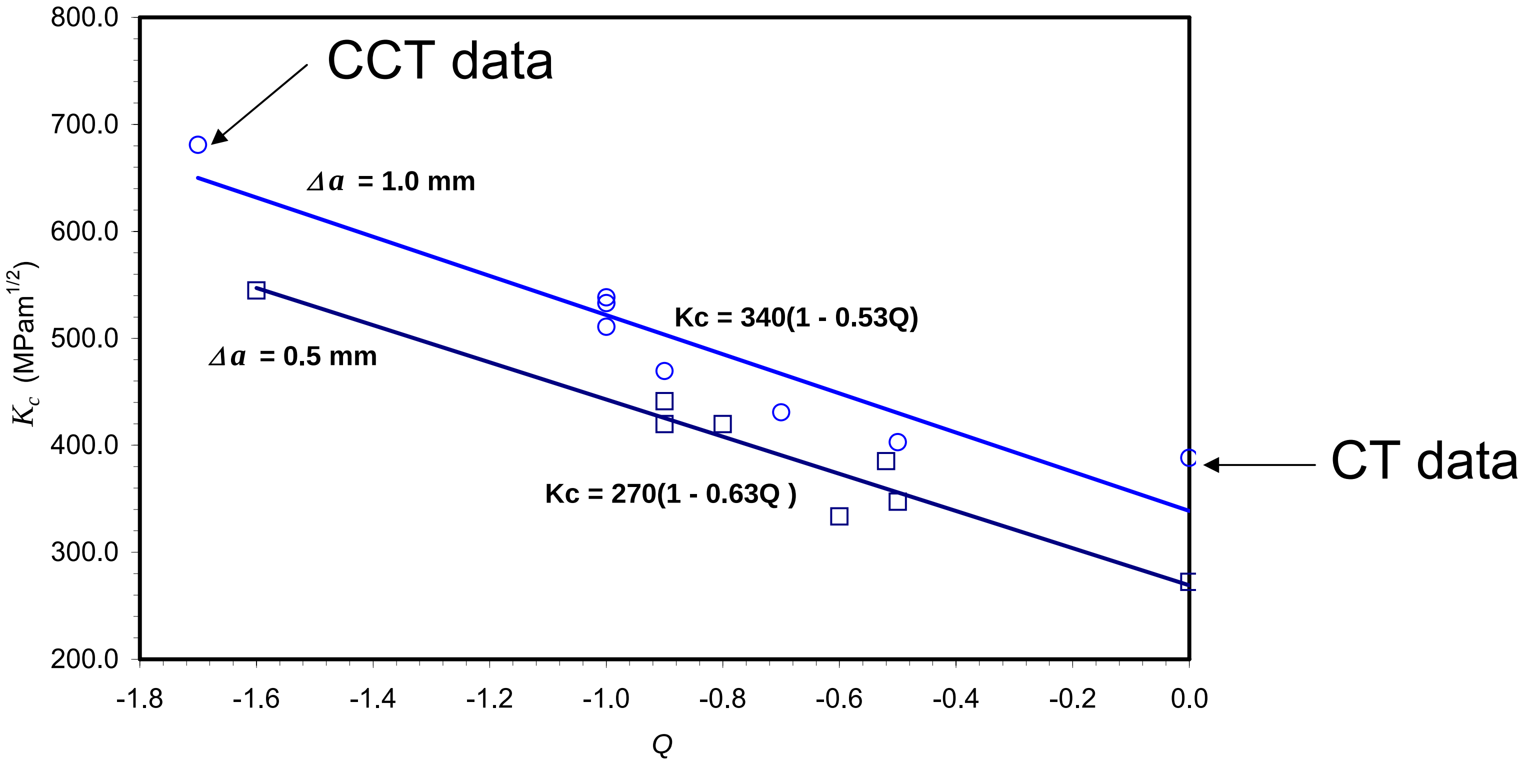
Crack tip mesh



- Full 3D analysis

10.75” pipe; X65
(406.4 mm × 19.1 mm)

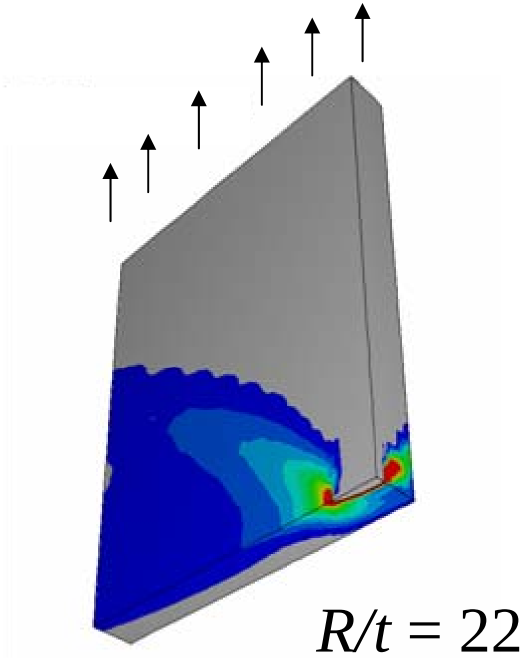
Approach for Steel Pipelines, X100

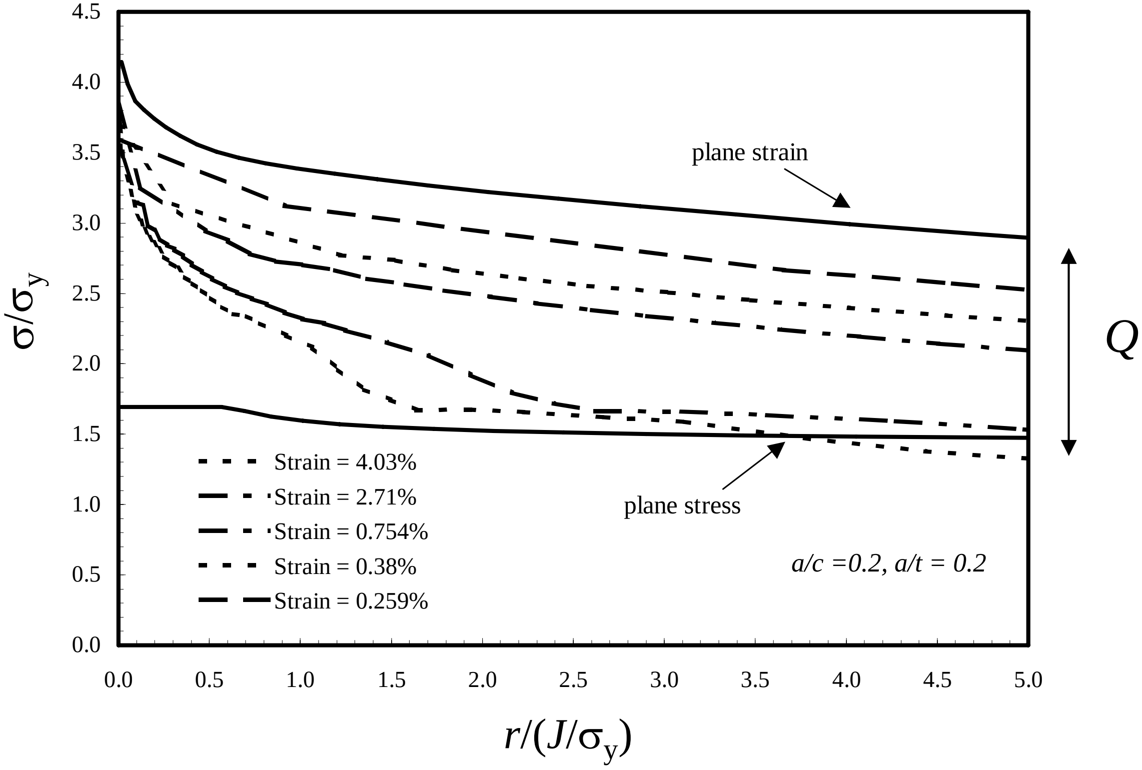


O’Dowd and McGillivray, 2003

- *J*-*Q* curve determined from testing small specimens

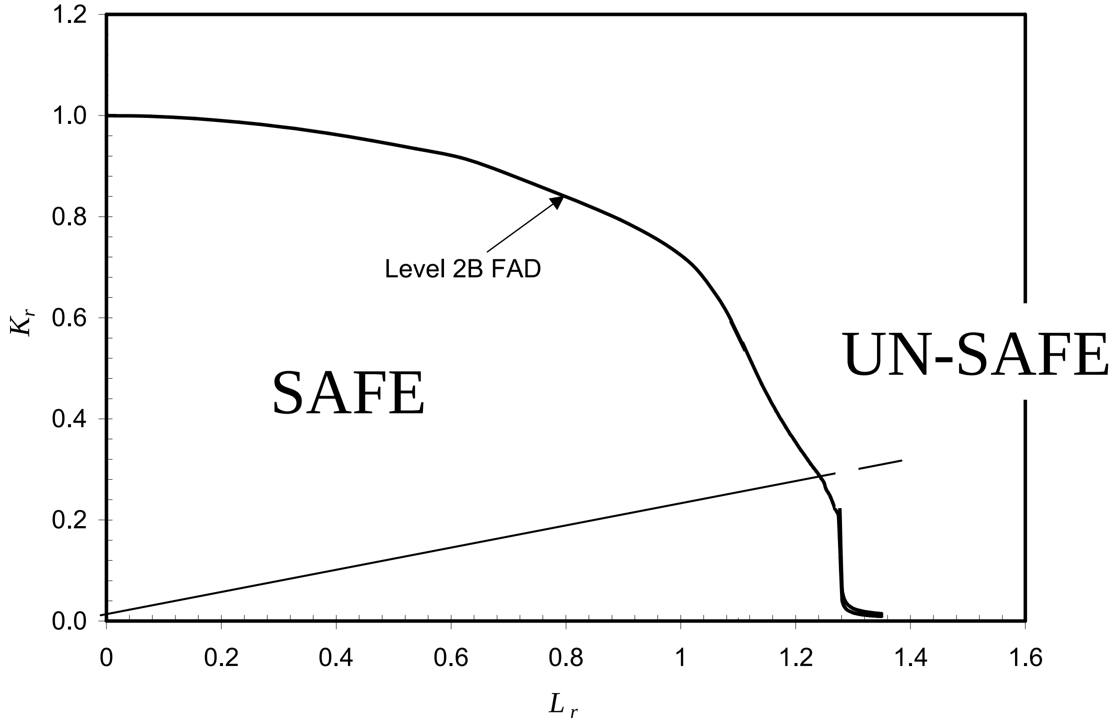
Approach for Steel Pipelines, X100





- Constraint variation in pipe from a 3D finite element analysis

Approach for Steel Pipelines, X100



- Additional safety margin from a constraint based analysis

Conclusions

- Crack tip driving force for an elastic-plastic material can be described by two parameters, J and Q
- Q is a hydrostatic stress term, motivated by the form of the crack tip fields
- Allowance for constraint can increase the safety margin or increase allowable load